Schrodinger eqn. for Hydrogen atom.

Substituting (8) in (7)

$$\frac{1}{R}\frac{d}{\partial r}\left(r^2\frac{dR}{dr}\right)+\frac{1}{\Theta\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{d\Theta}{d\theta}\right)+\frac{2mr^2}{h^2}\left(E+\frac{e^2}{4\pi\epsilon_0 r}\right)=\frac{m_l^2}{\sin^2\theta}$$

segregating functions of $r$ and $\theta$

$$\frac{1}{R}\frac{d}{\partial r}\left(r^2\frac{dR}{dr}\right)+\frac{2mr^2}{h^2}\left(E+\frac{e^2}{4\pi\epsilon_0 r}\right)=\frac{m_l^2}{\sin^2\theta}-\frac{1}{\Theta\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{d\Theta}{d\theta}\right) \quad \text{— (8a)}$$

The two sides of the equation can be equal only if they have a common value

This equality requires that both sides are equal to a constant $l^2+l$ or $l(l+1)$

$$\frac{1}{R}\frac{d}{\partial r}\left(r^2\frac{dR}{dr}\right)+\frac{2mr^2}{h^2}\left(E+\frac{e^2}{4\pi\epsilon_0 r}\right)=l(l+1)$$

Dividing throughout by $r^2$ and multiplying by $R$ and rearranging

$$\frac{1}{r^2}\frac{d}{\partial r}\left(r^2\frac{dR}{dr}\right)+\left[\frac{2m}{h^2}\left(E+\frac{e^2}{4\pi\epsilon_0 r}\right)-\frac{l(l+1)}{r^2}\right]R=0 \quad \text{— (9)}$$

From eqn (8a)

$$\frac{m_l^2}{\sin^2\theta}-\frac{1}{\Theta\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{d\Theta}{d\theta}\right)=l(l+1)$$

multiplying throughout by $\Theta$

$$\frac{1}{\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{d\Theta}{d\theta}\right)+\left[l(l+1)-\frac{m_l^2}{\sin^2\theta}\right]\Theta=0 \quad \text{— (10)}$$

So we obtained three main equations

$$\frac{d^2\phi}{d\phi^2}+m_l^2\phi=0 \quad \text{— (8)}$$

$$\frac{1}{r^2}\frac{d}{\partial r}\left(r^2\frac{dR}{dr}\right)+\left[\frac{2m}{h^2}\left(E+\frac{e^2}{4\pi\epsilon_0 r}\right)-l(l+1)\right]R=0 \quad \text{— (9)}$$

$$\frac{1}{\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{d\theta}{d\theta}\right)+\left[l(l+1)-\frac{m_l^2}{\sin^2\theta}\right]\Theta=0 \quad \text{——— (10)}$$

Now

$$\frac{d^2\phi}{d\phi^2}+m_l^2\phi=0$$

upon solving the eqn.

$$\phi = Ae^{im_l\phi} \quad \text{——— (a)}$$

where A is the constant of integration and R.H.S of the equation is a periodic function

i.e $f(x) = f(x+t) = f(x+2t)$ i.e. the values repeat themse after certain intervals of time

$$\phi(\phi) = \phi(\phi+2\pi) \quad \text{——— (b)}$$

This function is repeating itself after $2\pi$

substituting b in a

$$Ae^{im_l\phi} = Ae^{im_l(\phi+2\pi)}$$

This eqn is true when $m_l$ is either zero or an integer

$$m_l = 0, \mp1, \mp2, \mp3, \mp4 \ldots$$

This $m_l$ is magnetic quantum no.

upon solving this differential equation

$$\frac{1}{\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{d\theta}{d\theta}\right)+\left[l(l+1)-m_l^2\right]\Theta=0 \quad \text{——— (10)}$$

we deduce that $l$ may be an integer equal to or greater than $|m_l|$

$$m_l = 0, \mp1, \mp2, \mp3, \mp4 \ldots \mp l$$

where $l$ is the azimuthal quantum number

upon solving this differential equation

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{dR}{dr}\right)+\left[\frac{2m}{h^2}\left(E+\frac{e^2}{4\pi\epsilon_0 r}\right)-l(l+1)\right]R=0 \quad \text{——— (9)}$$

we get the formula for energy

$$E_n = \frac{-me^4}{32\pi^2\epsilon_0^2\hbar^2n^2} \quad \text{where } n = 1,2,3,4 \ldots$$

Energy can have any positive value but negative value are quantized, so quantization condition applies only on negative values.

The formula obtained by Bohr for energy of electron is exactly the same as obtained by Schrodinger

This $n$ is a constant called principal quantum no, and its value must be equal to or greater than $(l+1)$

$$l = 0, 1, 2, 3 \; --- \; (n-1)$$

why didn't spin quantum no. appear in this

The reason is that when we started this equation our main concern was finding the position of electron so we neglected the spin of the electron from the beginning.